absorbers, and has the advantage of posing no circuit reliability problems.

The results presented were limited to the TM_{110} mode. The analysis may be easily applied to higher order cavity modes if the circuit couples to those modes.

References

[1] B R. Hallford and C. E Bach, "Lid interaction protected shield enclosed dielectric mounted microstrip," U S. Patent 3638148, Jan. 25, 1972.

[2] J. R. Nortier, C. A. Van der Neut, and D. E. Baker, "Tables for the design of The Jaumann microwave absorber," *Microwave J.*, vol. 30, no. 9, pp. 219–222, Sept. 1987.

[3] R F Harrington, *Time Harmonic Electromagnetic Fields*. New York: McGraw–Hill, 1961, pp. 158–163

[4] R. E. Collin, *Foundations for Microwave Engineering*. New York: McGraw–Hill, 1966, ch. 3

[5] S. Ramo, J. R. Whinnery, and T Van Duzer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1984, ch. 5.

Finite Element Formulation for Guided-Wave Problems Using Transverse Electric Field Component

KAZUYA HAYATA, MASASHI EGUCHI, AND MASANORI KOSHIBA, SENIOR MEMBER, IEEE

***Abstract* — A finite-element formulation for electromagnetic waveguide problems is described using the transverse electric field component. In this approach, the divergence relation $\nabla \cdot \boldsymbol{D} = 0$ is satisfied and spurious solutions can be eliminated in the entire region of a propagation diagram. The validity of the formulation is examined via applications to a few canonical guided-wave problems.**

I. Introduction

The most serious difficulty in applying the finite element method to waveguide problems has been the appearance of so-called spurious, nonphysical solutions. To overcome this difficulty, various approaches have recently been developed; these are reviewed in [1]. More recently, a new finite element formulation for the analysis of dielectric waveguide modes has been developed by the authors in terms of the transverse magnetic field component ($\boldsymbol{H}_t$) [2]. The key point of this method, which is distinctly different from other transverse field methods [3]–[7], is that it transforms the finite element equation in terms of a full vector $\boldsymbol{H}$ field [8], [9] into one in terms of only the transverse magnetic field component, using the condition $\nabla \cdot \boldsymbol{H} = 0$. In this approach, the spurious solutions can be completely eliminated in the entire region of a propagation diagram, and the final matrix dimension is reduced to two thirds that of the conventional three-component approach using the penalty function method [10]–[14]. However, in the finite element analysis based on this approach, the magnetic field components are first obtained as an eigenvector, and the electric field components are later derived from them via Maxwell's equations. This additional operation based on spatial differentiations of the original data may cause an unnatural field profile when one uses lower order Lagrange elements.

In this paper, as an electric field version of the method described in [2], a finite element method for electromagnetic waveguide problems is formulated using the transverse electric field component ($\boldsymbol{E}_t$). In this approach, the spurious solutions can be eliminated in the entire region of a propagation diagram, and the dimension of the final matrix equation is reduced to two thirds that of the full vector $\boldsymbol{E}$ field approach based on a penalty function [15], [16]. The validity of the present method is confirmed via applications to a few representative waveguiding problems.

Manuscript received March 18, 1988; revised June 20, 1988.

The authors are with the Department of Electronic Engineering, Hokkaido University, Sapporo, 060 Japan.

IEEE Log Number 8824248.

II. Formulation

We consider a waveguide with a tensor permeability and a scalar permittivity. With a time dependence of the form $\exp(j\omega t)$ being implied, from Maxwell's equations the following wave equation is derived:

$$\nabla \times \left([\mu]^{-1} \nabla \times \boldsymbol{E}\right) - k_0^2 \epsilon \boldsymbol{E} = 0 \tag{1}$$

where ω is the angular frequency, k_0 is the free-space wavenumber, $[\mu]$ is the relative permeability tensor, and ϵ is the relative permittivity, which is assumed to be constant in each material.

The divergence relation for source-free media, $\nabla \cdot \boldsymbol{D} = 0$, can be written

$$\epsilon E_z = (j\beta)^{-1}\left(\epsilon \partial E_x/\partial x + \epsilon \partial E_y/\partial y\right) \tag{2}$$

where β is the phase constant in the propagation direction (z direction).

Application of the standard finite element technique [2] to (1) and (2) gives the following matrix equations:

$$[S]\{E\} - (k_0/\beta)^2[T]\{E\} = \{0\} \tag{3}$$

$$[D_z]\{E_z\} = [D_t]\{E_t\} \tag{4}$$

where

$$[S] = \sum_e \iint_e [B]^*[\mu]_e^{-1}[B]^T \, d\bar{x}\, d\bar{y} \tag{5}$$

$$[T] = \sum_e \iint_e \epsilon_e [N]^*[N]^T \, d\bar{x}\, d\bar{y} \tag{6}$$

$$[D_z] = \sum_e \iint_e \epsilon_e \{N\}\{N\}^T \, d\bar{x}\, d\bar{y} \tag{7}$$

$$[D_t] = -\sum_e \iint_e \left[\epsilon_e \{N\}\{N\}_x^T \quad \epsilon_e \{N\}\{N\}_y^T\right] d\bar{x}\, d\bar{y} \tag{8}$$

$$\{E_t\} = \begin{bmatrix} \{E_x\} \\ \{E_y\} \end{bmatrix}. \tag{9}$$

Here, $\{N\}$ is the shape function vector; $\{0\}$ is a null vector; T, $\{\cdot\}$, and $\{\cdot\}^T$ denote a transpose, a column vector, and a row vector, respectively; the components of vectors $\{E_x\}$, $\{E_y\}$, and $\{E_z\}$ are the values of E_x, E_y, and E_z at nodal points in the cross section, respectively; * denotes complex conjugate; $\bar{x} = \beta x$ and $\bar{y} = \beta y$; and $[N]$ and $[B]$ are given in [14].

Using (4), we can express the nodal electric field vector $\{E\}$ in terms of $\{E_t\}$:

$$\{E\} = [D]\{E_t\} \tag{10}$$

where

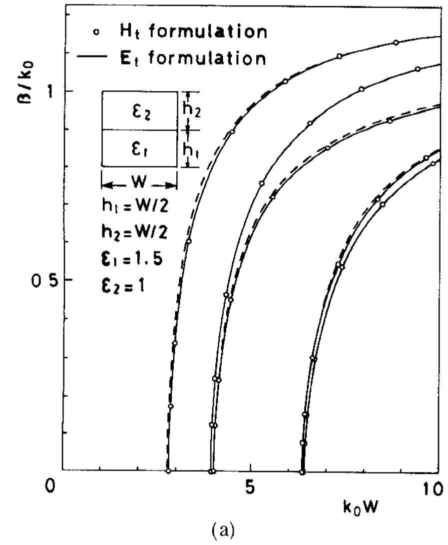
$$[D] = \begin{bmatrix} [U] \\ [D_z]^{-1}[D_t] \end{bmatrix}. \tag{11}$$

Here $[U]$ is a unit matrix.

Substituting (10) into (3) and multiplying (3) by $[D]^T$ from the left, we obtain the following final matrix equation with respect to the transverse electric field component $\{E_t\}$:

$$[\tilde{S}_{tt}]\{E_t\} - (k_0/\beta)^2[\tilde{T}_{tt}]\{E_t\} = \{0\} \tag{12}$$

0018-9480/89/0100-0256$01.00



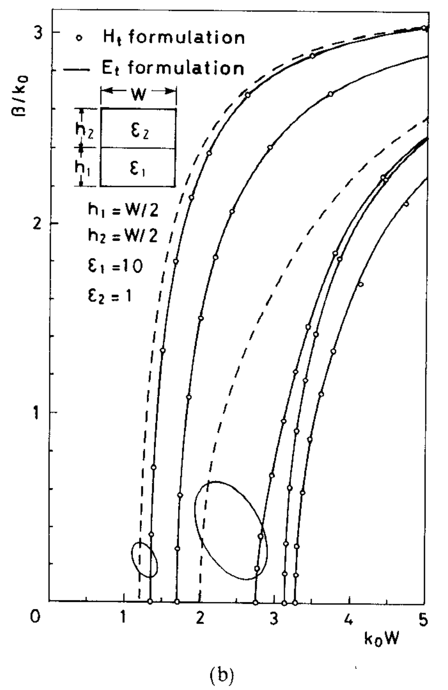


Fig. 1. Dispersion characteristics of half-filled dielectric waveguides. (a) $\epsilon_1 = 1.5$. (b) $\epsilon_1 = 10$.

where

$$[\tilde{S}_{tt}] = [D]^T[S][D] \tag{13}$$

$$[\tilde{T}_{tt}] = [D]^T[T][D]. \tag{14}$$

In (12), the nodal electric field vector should be forced to satisfy the boundary conditions at the interface between two media with different permittivities [15], [16]. We consider an interface Γ with an abrupt discontinuity in the permittivity as shown in [15, fig. 1]. The tangential components of $\boldsymbol{E}$ and the normal component of $\epsilon \boldsymbol{E}$ should be continuous at the interface Γ. These boundary conditions can be written as

$$\{E_x\}_2 = q_{xx}\{E_x\}_1 + q_{xy}\{E_y\}_1 \tag{15}$$

$$\{E_y\}_2 = q_{xy}\{E_x\}_1 + q_{yy}\{E_y\}_1 \tag{16}$$

$$\{E_z\}_2 = \{E_z\}_1 \tag{17}$$

where the components of $\{E_t\}_i$ are the values of E_t at the nodal points on Γ included in the element e_i. The quantities q_{xx}, q_{xy},

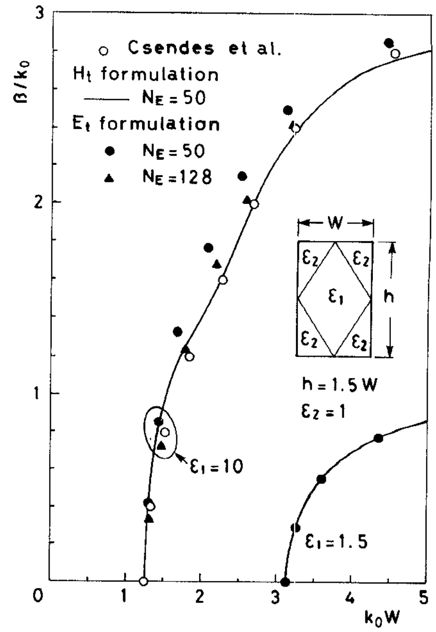


Fig. 2. Dispersion characteristics of rectangular waveguide with diamond-shaped insert.

and q_{yy} are given by

$$q_{xx} = \sin^2\theta + (\epsilon_1/\epsilon_2)\cos^2\theta \tag{18}$$

$$q_{xy} = (\epsilon_1/\epsilon_2 - 1)\sin\theta\cos\theta \tag{19}$$

$$q_{yy} = \cos^2\theta + (\epsilon_1/\epsilon_2)\sin^2\theta \tag{20}$$

where ϵ_1 and ϵ_2 are the relative permittivities of regions 1 and 2, respectively, and θ is the angle between the unit vector normal to Γ and the x axis.

By using the original functional for e_1 and the modified functional for e_2, which can be obtained by considering (15)–(17) in the original one, the boundary conditions of the electric field $\boldsymbol{E}$ at the interface with an abrupt discontinuity in the permittivity are satisfied [15].

Equation (12) is an ordinary matrix eigenvalue problem whose eigenvalue and eigenvector are $(k_0/\beta)^2$ and $\{E_t\}$, respectively. It should be noted that the divergence condition $\nabla \cdot \boldsymbol{D} = 0$ is implicitly included in (12) and the matrix dimension is reduced to two thirds that of the penalty function method [15], [16]. The electric field can be obtained first as an eigenvector in (12), and the tensor permeability is considered. (Note that a constant scalar permeability is assumed in [2].)

III. Numerical Examples

In this section, we present computed results obtained by the present formulation. In numerical computations, double precision is used to avoid roundoff errors and Householder's method is used as an eigenvalue solution method. Since the matrices (13) and (14) are dense, unlike those generated by the penalty approach, there is no possibility of taking advantage of sparsity with a sophisticated eigenvalue solver. An arbitrary small value of the input datum, e.g., $\beta W = 10^{-3}$, 10^{-2}, virtually gives a cutoff value of $k_0 W$.

Fig. 1(a) and (b) shows the dispersion characteristics for the first five modes of half-filled dielectric waveguides, where the plane of symmetry is assumed to be a perfect magnetic conductor and one half of the cross section is divided into second-order triangular elements (number of elements $(N_E) = 36$, number of nodal points $(N_p) = 91$). In Fig. 1, the broken lines correspond to the case in which no additional boundary conditions are explicitly imposed at the interface between media with different permittivities. It is found from Fig. 1 that the agreement is very good between the results obtained by the present formulation (solid lines) and those obtained by the formulation using the transverse magnetic field component (hollow circles) [2]. The spurious solutions do not appear in the entire region of a propagation diagram.

As a more complicated waveguiding configuration, we next consider the rectangular waveguide with a diamond-shaped insert studied by Csendes and Silvester [17], as illustrated in Fig. 2. In this waveguide, there are abrupt changes in the permittivity at the interface, the normal direction of which does not coincide with the direction of a coordinate axis. Fig. 2 shows the dispersion characteristics for the fundamental mode, where two planes of symmetry are assumed to be perfect magnetic conductors and one quarter of the cross section is divided into second-order triangular elements. In Fig. 2, the results of the $\boldsymbol{H}_t$ field formulation [2] with $N_E = 50$ and $N_p = 121$ and those of the modal approximation technique [17] are also presented. For $\epsilon_1 = 1.5$, the results of the present $\boldsymbol{E}_t$ field formulation with $N_E = 50$ and $N_p = 121$ agree well with those of the $\boldsymbol{H}_t$ counterpart. On the other hand, for a larger value of relative permittivity, $\epsilon_1 = 10$, the results of the $\boldsymbol{E}_t$ field formulation with $N_E = 50$ deviate from those of the $\boldsymbol{H}_t$ counterpart at higher frequencies. This deviation at higher frequencies reflects the singularity of the normal electric field component at the tips of wedges of the dielectric insert [18]–[20]. Such a singularity near the tip of a dielectric corner may cause electrical breakdown in high-power applications. It is evident from Fig. 2 that the $\boldsymbol{E}_t$ field finite element solutions can be improved by increasing the number of elements. Indeed, the results of the $\boldsymbol{E}_t$ field formulation with $N_E = 128$ and $N_p = 289$ are closer to those of the $\boldsymbol{H}_t$ field counterpart. No spurious solutions are involved in this case as well.

IV. Conclusions

We have formulated a vectorial finite element scheme for solving guided-wave problems using the transverse electric field component. Considering the duality between electric and magnetic field vectors in Maxwell's equations, we have used the same procedure as the transverse magnetic field counterpart except that additional conditions are enforced on the boundary between different dielectric materials. In this approach, the electric field components can be directly obtained as an eigenvector of a matrix eigenvalue problem. Furthermore, no spurious solutions are involved in the entire region of a propagation diagram, and the dimension of the final matrix equation is reduced to two thirds that of the penalty function method. We have confirmed the validity of the present formulation via applications to some canonical waveguide problems.

The approach described in this paper is also applicable to waveguides containing anisotropic media such as ferrites because the tensor permeability may vary from material to material. Furthermore, extension to waveguides containing lossy and/or active media is straightforward if one uses the procedure that has recently been proposed by the authors [21] for the magnetic field case.

References

[1] M. Koshiba, K. Hayata, and M. Suzuki, "Finite-element method analysis of microwave and optical waveguides—Trends in countermeasures to spurious solutions," *Trans. Inst. Electron. Commun. Eng. Japan*, vol. J69-C, pp. 1477–1486, Dec. 1986 (in Japanese); to be published in English in *Electron. Commun. Japan*.

[2] K. Hayata, M. Koshiba, M. Eguchi, and M. Suzuki, "Vectorial finite-element method without any spurious solutions for dielectric waveguiding problems using transverse magnetic-field component," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1120–1124, Nov 1986.

[3] D. Gelder, "Numerical determination of microstrip properties using the transverse field components," *Proc. Inst. Elec. Eng.*, vol. 117, pp. 699–703, Apr. 1970.

[4] C. G Williams and G. K Cambrell, "Numerical solution of surface waveguide modes using transverse field components," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 329–330, Mar. 1974.

[5] K Bierwirth, N. Schulz, and F Arndt, "Finite-difference analysis of rectangular dielectric waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol MTT-34, pp 1104–1114, Nov. 1986.

[6] C.-C. Su, "A combined method for dielectric waveguides using the finite-element technique and the surface integral equations method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1140–1146, Nov. 1986.

[7] T. Angkaew, M. Matsuhara, and N. Kumagai, "Finite-element analysis of waveguide modes: A novel approach that eliminates spurious modes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 117–123, Feb. 1987.

[8] A. Konrad, "High-order triangular finite elements for electromagnetic waves in anisotropic media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 353–360, May 1977

[9] B M. A. Rahman and J B. Davies, "Finite-element analysis of optical and microwave waveguide problems," *IEEE Trans. Microwave Theory Tech.*, vol MTT-32, pp. 20–28, Jan. 1984.

[10] M. Koshiba, K. Hayata, and M Suzuki, "Vectorial finite-element formulation without spurious modes for dielectric waveguides," *Trans. Inst. Electron. Commun. Eng. Japan*, vol. E67, pp. 191–196, Apr. 1984.

[11] M. Koshiba, K. Hayata, and M. Suzuki, "Vectorial finite-element method without spurious solutions for dielectric waveguide problems," *Electron. Lett.*, vol 20, pp. 409–410, May 1984.

[12] B. M A. Rahman and J. B. Davies, "Penalty function improvement of waveguide solution by finite elements," *IEEE Trans. Microwave Theory Tech.*, vol MTT-32, pp. 922–928, Aug. 1984.

[13] M. Koshiba, K. Hayata, and M Suzuki, "Study of spurious solutions of finite-element method in the three-component magnetic-field formulation for dielectric waveguide problems," *Electron. Commun. Japan*, pt. 1, vol. 68, pp. 114–119, Aug. 1985.

[14] M. Koshiba, K. Hayata, and M. Suzuki, "Improved finite-element formulation in terms of the magnetic-field vector for dielectric waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp 227–233, Mar. 1985.

[15] M. Koshiba, K. Hayata, and M. Suzuki, "Finite-element formulation in terms of the electric-field vector for electromagnetic waveguide problems," *IEEE Trans Microwave Theory Tech.*, vol. MTT-33, pp. 900–906, Oct. 1985

[16] M. Koshiba, K. Hayata, and M. Suzuki, "Vector *E*-field finite-element analysis of dielectric optical waveguides," *Appl. Opt.*, vol 25, pp 10–11, Jan 1986.

[17] Z. J. Csendes and P. Silvester, "Numerical solution of dielectric loaded waveguides: II—Modal approximation technique," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 504–509, June 1971.

[18] J. Meixner, "The behavior of electromagnetic fields at edges," *IEEE Trans. Antennas Propagat.*, vol. AP-20, pp 442–446, July 1972

[19] J B. Andersen and V V. Solodukhov, "Field behavior near a dielectric wedge," *IEEE Trans Antennas Propagat.*, vol. AP-26, pp. 598–602, July 1978.

[20] J V. Bladel, "Field singularities at metal-dielectric wedges," *IEEE Trans. Antennas Propagat.*, vol. AP-33, pp. 450–455, Apr. 1985.

[21] K Hayata, K. Miura, and M. Koshiba, "Finite-element formulation for lossy waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 268–276, Feb. 1988.

An Improved Algorithm for the Computer-Aided Design of Coupled Slab Lines

STANISŁAW ROSŁONIEC

***Abstract* — Improved analytical formulas for the computer-aided design of parallel coupled slab lines are presented. These formulas ensure a good agreement of calculations with accurate numerical results for a wide range of geometrical dimensions of the lines.**

Manuscript received March 21, 1988; revised July 12, 1988.
The author is with the Institute of Radioelectronics, Warsaw Technical University, 00-665 Warsaw, Nowowiejska 15/19, Poland
IEEE Log Number 8824251

0018-9480/89/0100-0258$01.00 ©1989 IEEE